Lecture 25: Revision

Statistics 251

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Lecture Outline

Probability Space

Random Variables
- Discrete Random Variables
- Continuous Random Variables

Operations of Random Variables

Estimation and Limit Theorems

Where are we?

Probability Space

Random Variables
- Discrete Random Variables
- Continuous Random Variables

Operations of Random Variables

Estimation and Limit Theorems

Sample Space

- Sample (state), sample space, event
- Union, Intersection, Mutually exclusive, Complement
- Axioms of probability: bound, whole space, addition
- calculation of probability, $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$, several events.
- Conditional Probability, Bayes' rule ($\mathbb{P}(B \mid A) \cdot \mathbb{P}(A) = \mathbb{P}(A \cap B) = \mathbb{P}(A \mid B) \cdot \mathbb{P}(B)$), independence of events

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Random Variables

- Random variable is a function from sample to real numbers
- probability mass function, probability distribution function, cumulative distribution function
- joint distribution, independent rv.
- expectation, expectation of functionals, affine transform, sample mean
- variance, standard derivation, covariance, correlation, affine transform

Discrete Random Variables

- pmf, mean, variance
- Indicator function = Bernoulli, binomial, Poisson, Geometric, negative binomial

Given we have an independent Bernoulli test in each time-slot.

- Binomial r.v. is the total count of success within some interval
- Poisson r.v. is limit of binomial distribution given mean of count of success fixed.
- Geometric r.v. is count of test until the first success
- Negative Binomial r.v. is the count of test until several success.

Continuous Random Variables

pdf, mean, variance, normalizing factor

- Gaussian (normal), bivariate, scaling and centering, summation,
- Exponential, scaling, min of independent exp, memoryless
- Gamma, is generalized exp, scaling, summation
- Cauchy distribution, no mean

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Operations of Random Variables

- Change of Variables: $Y = g(X)$, $f_Y(y) = f_X(x)|J_G(x)|^{-1}$
- Summation (convolution): $f_{X+Y}(a) = \int_{-\infty}^{\infty} f_X(x) f_Y(a-x) dx$
- Conditional distribution (by event, by rv), conditional expectation: $f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)}$
- Moment Generating Functions: $M(t) = \mathbb{E}\left[e^{tX}\right]$, summation, Gaussian

Where are we?

Estimation and Limit Theorems

- Markov inequalities, Chebyshev inequality

$$\mathbb{P}(X \geq a) \leq \frac{\mathbb{E}[X]}{a}, \quad \mathbb{P}\Big(|X-\mu| \geq k\Big) \leq \frac{\sigma^2}{k^2}$$

- weak and strong Law of Large Numbers

$$P\left\{\left|\frac{X_1+\cdots+X_n}{n}-\mu\right| \geq \varepsilon\right\} \to 0 \quad \text{as} \quad n \to \infty$$

$$\lim_{n\to\infty} \frac{X_1+\cdots+X_n}{n} = \mu \quad \text{with probability 1}$$

- Central Limit Theorem

$$\frac{X_1+\cdots+X_n-n\mu}{\sigma\sqrt{n}} \to N(0,1) \quad \text{in distribution}$$