Lecture 1: Counting Principles

Statistics 251

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Where are we?

Course Overview

Why study probability?

Principles of counting

Problems and examples

Welcome to Stat 251!

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Static and summarized information is on the course homepage:
http://www.stat.uchicago.edu/~zhongjian/stat251_2020aut/CourseHomepage.html
(Too long? You can find a clickable one on Canvas.)

- Course questions: post it on Canvas Discussion
- Weekly homeworks due Monday by lecture
  - Submit on Gradescope (find invitation email this afternoon)
  - No late HW accepted
- Midterm on October 26, Final December 9-11
- Grade assigned based on maximum of
  - 20% HW + 80% Final
  - 20% HW + 30% Midterm + 50% Final

Course Outline

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1. Combinatorial analysis
2. Probability spaces and conditional probability
3. Discrete random variables
4. Continuous random variables
5. Change of variables
6. Limit theorems (Law of large numbers, Central limit theorem)

Where are we?

Disease prevalence

Suppose 50% of the population has a disease.

- In a group of 1 person, how many people should you expect have the disease?
- In a group of 100 people, how many people should you expect have the disease?
- In a group of 10000 people, how many people should you expect have the disease?

Suppose you test a group of N people and they all have the disease. How surprising is this for..

- $N = 1$?
- $N = 10$?
- $N = 100$?

Disease prevalence

Suppose 1% of the population has a disease.

- In a group of 1 person, how likely is it that someone has the disease?
- In a group of 100 people, how likely is it that someone has the disease?
- In a group of 10000 people, how likely is it that someone has the disease?

How do the answers change if 1% becomes 2%?

Where are we?

Addition and Multiplication Principles

Addition principle: Add up disjoint groups

- Q: If a class has 10 freshmen, 4 sophomores, 15 juniors, and 10 seniors, how many ways can we choose a project with only one member?

Multiplication principle: If independent choices are made, multiply the possible values of those choices.

- Q: If a class has 10 freshmen, 4 sophomores, 15 juniors, and 10 seniors, how many ways can we choose a project group with one member from each class?

Counting subsets

How many subsets does a set of size n have? Note: In Math/Prob world, all elements in any one set are disjoint!

Permutations

How many ways are there to order a deck of 52 different cards?

If there are n people and n hats, how many ways are there to assign each person a hat?

Permutations

A **permutation** is a way to reorder a set of n distinct objects. The number of such permutations is

$$n! = n \cdot (n-1) \cdots 2 \cdot 1.$$

If there are $k < n$ people and n hats, how many ways are there to assign each person a hat?

Overcounting

How many ways are there to rearrange the letters in the word *TEA*?

How many ways are there to rearrange the letters in the word *GOOD*?

Overcounting

Instead of counting the size of a set directly, it can be useful to **overcount by a fixed factor** and divide by that factor.
You should derive and double-check the factor applies to all cases!

How many ways can a desk of indistinguishable 3 black, 4 red cards be shuffled?

Combinations

The **binomial coefficient**

$$\binom{n}{k} := \frac{n!}{k!(n-k)!}$$

is the number of ways to choose a set of k items from n distinguishable items.

- $\frac{n!}{(n-k)!} = n \cdot (n-1) \cdots (n-k+1)$ is the number of ways to choose k **ordered** items
- This overcounts by $k!$, so we divide to get $\binom{n}{k}$.

Binomial Theorem

What is the coefficient in front of x^k in the expansion of $(x+1)^n$?

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The **binomial theorem** says that

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = x^n + \binom{n}{1} x^{n-1} y + \cdots + \binom{n}{n-1} x y^{n-1} + y^n.$$

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Some kind notes:

- Not all problems have elegant solutions, e.g. existence of overcount factor.
- The reason that a person can quickly solve one counting problem is always because he/she has already known the way to solve for that problem or similar problems before.
- So Never asking for answers, before you have spent more than half an hour when facing a new problem.

Counting problems

How many ways can 10 identical beads be divided between 3 distinguishable cups?

Counting numbers

How many ways to assign a birthday to each of 19 distinct people?
What if no birthday can be repeated?

Game time:) Go to Canvas Discussion and find the link!

Counting poker hands

In a poker game, there are 52 playing cards (Rank: A,K,Q,J,9,8,7,6,5,4,3,2. $\times$ Suit: Spade Diamond Heart Club). A hand refers to a set of five cards.
How many poker hands are there?

Counting poker hands

A full house hand refers to a set of cards with 3 cards in one rank and 2 cards in another rank. How many full house hands can one make in poker?

Counting poker hands

How many 2 pair hands can one make in poker?

Counting poker hands

How many hands have 4 cards of one suit and 1 card of another suit?

Counting numbers

How many 10 digit numbers have no consecutive digits the same?

Counting problems

How many ways are there to divide a class of 60 students into 3 groups of 20?