Statistics 251 Name: ______________________________
Section 1, Autumn 2020
Midterm Exam (Example) CNetID: ______________________________
October 26, 2020
Time 10:20 AM-11:10 AM CDT, accept submission until 11:30 AM

Instructions: This exam contains 4 problems. Please make sure you attempt all problems.

Present your solutions in a **legible, coherent** manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Unsupported or illegible solutions may not receive full credit.

You may not print this test paper out. In that case, you should sign at the beginning of you uploaded document to affirm you have followed all the rules. **You should also draw a box at start of each question with your final answer**. An example is shown as below, *slanted* sentences are ones to be replaced with your own anwser.

Name: *(your full name in block letters)* **CNetID:** *(your CNetID)*
I am here to affirm that I have followed all the rules listed on the original midterm exam paper.
(your signature and the date)

Problem 1 (a) Answer: *(write your final answer here)*

Your solution/explanation to the final answer should follow the box.

Problem 1 (b) Answer: *(write your final answer here)*

Your solution/explanation to the final answer should follow the box.

Due to the coronavirus situation, this exam will be over Zoom. The use of outside material including books, notes, and electronic smart devices is not allowed. You are not allowed to discuss anything about this midterm with anybody (in this session or in the parallel session or not in the course) except your instructor until **Oct. 26 11:59 PM CDT**. You may chat with the instructor in the Zoom room during the test. Only when you have successfully uploaded your answer, you are allowed to leave the Zoom meeting.

Please sign below to affirm that you have followed these rules.

Signature: ______________________________

Formulas

Probability mass functions:

- Binomial with parameters n and p: $\mathbb{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$
- Poisson with parameter λ: $\mathbb{P}(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$
- Geometric with parameter p: $\mathbb{P}(X = k) = p(1-p)^{k-1}$
- Negative binomial with parameters r and p: $\mathbb{P}(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$

Problem 1 (10 points) How many quintuples $(a_1, a_2, a_3, a_4, a_5)$ of non-negative integers satisfy $a_1 + a_2 + a_3 + a_4 + a_5 = 100$?

Answer:

Problem 2 (20 points) Thirty people are invited to a party. Each person accepts the invitation, independently of all others, with probability $1/3$. Let X be the number of accepted invitations. Compute the following:

(a) (5 points) $\mathbb{E}[X]$

Answer:

(b) (5 points) $\mathrm{Var}(X)$

Answer:

(c) (5 points) $\mathbb{E}[X^2]$

Answer:

(d) (5 points) $\mathbb{E}[X^2 - 4X + 5]$

Answer:

Problem 3 (20 points) Bob has noticed that during every given minute, there is a 1/720 chance that the Facebook page for his dry cleaning business will get a like, independently of what happens during any other minute. Let L be the total number of likes that Bob receives during a 24 hour period.

(a) (5 points) Compute $\mathbb{E}[L]$ and $\mathrm{Var}(X)$.

Answer:

(b) (5 points) Compute the probability that $L = 0$.

Answer:

(c) (10 points) Use a Poisson approximation to approximate the probability that $L \geq 2$.

Answer:

Problem 4 (10 points) Consider an infinite sequence of independent tosses of a coin that comes up heads with probability p. Compute (in terms of p) the probability that the fifth head occurs on the tenth toss.

Answer:

Problem 5 (20 points) A standard deck of 52 cards contains 4 aces. Suppose we choose a random ordering (all 52! permutations being equally likely). Compute the following:

(a) (5 points) The probability that all of the top 4 cards in the deck are aces.

Answer:

(b) (5 points) The probability that none of the top 4 cards in the deck is an ace.

Answer:

(c) (10 points) The expected number of aces among the top 4 cards in the deck.

Answer:

Problem 6 (20 points) There are ten children: five attend school A, three attend school B, and two attend school C. Suppose that a pair of two children is chosen uniformly at random from the set of all possible pairs of children. Let X be the number of students in the random pair that attend school A and let Y be the number in the pair that attend school B.

(a) (10 points) Compute $\mathbb{E}[XY]$.

Answer:

(b) (10 points) Given that the two children in this pair attend the same school, what is the conditional probability that they both attend school A?

Answer: