Math 18500,Winter 2022 - Weekly Goals
Instructor: Zhongjian Wang

Week 1.

(1) Given a first order equation $\frac{dy}{dx} = f(x, y)$, sketch its isoclines, slope field, and integral curves.
(2) Given an autonomous equation $\frac{dy}{dt} = f(y)$, find its equilibrium values and sketch its solutions.
(3) Given an equilibrium value of an autonomous equation, determine whether it is stable/unstable/semistable.
(4) Solve separable equations $\frac{dy}{dx} = f(x)g(y)$ by integration.

Week 2.

(1) Solve linear first order equations $\frac{dy}{dx} + p(x)y = f(x)$ using integrating factors.
(2) Find power series solutions of first and second order equations by repeated differentiation.
(3) Find power series solutions by substitution; identify recursion relations for the coefficients.
(4) Sketch the velocity field and solution curves of a system of first order equations. *

Week 3.

(1) Solve first order linear systems $\frac{d\vec{x}}{dt} = A\vec{x}$, when A has real eigenvalues.
(2) Solve first order linear systems $\frac{d\vec{x}}{dt} = A\vec{x}$, when A has complex eigenvalues.
(3) Sketch the solutions of $\frac{d\vec{x}}{dt} = A\vec{x}$ in both cases, by hand, with reasonable accuracy.
(4) Solve inhomogeneous equations $\frac{d\vec{x}}{dt} = A\vec{x} + \vec{b}$. *

Week 4.

(1) Solve second order homogeneous equations by exponential ansatz and superposition principle.
(2) Solve second order homogeneous equations in the "complex root" case.
(3) Solve second order equations by repeated integration. *
(4) Distinguish between oscillators which are underdamped, overdamped, and critically damped.

Week 5.

(1) Solve second order inhomogeneous equations by exponential ansatz and superposition principle.
(2) Solve second order inhomogeneous equations with sinusoidal forcing terms.
(3) Convert between the forms $\mathrm{Re}\left[Ze^{i\omega t}\right]$, $a\sin(\omega t) + b\cos(\omega t)$, and $A\cos(\omega t - \phi)$.
(4) Find the amplitude gain, phase shift, and resonant frequency of a sinuisoidally forced oscillator.

Week 6.

(1) Use Fourier series to solve second order linear ODE with periodic forcing terms.
(2) Find the Fourier series of a given function with a given period, in the real and complex forms.
(3) Find the Fourier series of odd and even functions efficiently by exploiting symmetry.
(4) Find sine series for functions on arbitrary intervals.

Week 7.

(1) Use Fourier series to solve PDE (e.g. the heat equation on an interval).
(2) Write a function as a Fourier integral by applying the Fourier transform and inversion formula. *
(3) Apply the rules for Fourier transforms and inverse Fourier transforms of derivatives. *
(4) Use Fourier transforms to solve PDE (e.g. the wave equation on the real line). *

Week 8.

(1) Find the Laplace transform of a given function.
(2) Find the inverse Laplace transform of a given function
(3) Solve ODE using Laplace transforms
(4) Find the inverse Laplace transform of a product.

Week 9.

(1) Represent arbitrary discontinuous functions in terms of step functions.
(2) Evaluate integrals and Laplace transforms involving step functions and delta functions.
(3) Solve ODE involving step functions and delta functions.
(4) Find the impulse response of an oscillator, and apply it to equations with arbitrary forcing terms.

* depends on actual teaching